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On overcoming problems in particle physics caused by ignorance of the structure of real space

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Abstract:
This chapter briefly reviews the main achievements of theoretical particle physics obtained using approaches based on symmetry and topology, and then highlights the weighty errors present in these theories. Particle theories are very abstract and developed within the framework of physical mathematics, which has no solid physical basis. However, real particles and real physical processes require finding a real basis on which fundamental physics should stand, and this is very important. The primary mathematical structure must be such that it is possible to apply mathematical physics to describe the physical picture of the world in real parameters that are actually present in particles and their interactions. It is precisely such a mathematical theory that is described in this work, the suitable geometry and topology are revealed, and references are provided to the corresponding physical theories developed by the author within the framework of mathematical physics, which already have a number of reliable experimental confirmations.

Keywords: particle physics; foundations of physics; real space; tessellattice; fractality

0.1 Introduction

In the Standard Model of particle physics, symmetry and topology are treated as fundamental concepts that describe how a quantum system's properties remain invariant under certain transformations. Symmetry refers to the rules and principles that govern a system's behaviour under changes to its coordinates, time, and/or quantum numbers. Topology operates with the intrinsic, geometric properties that are preserved under continuous deformations, such as stretching or bending, but not tearing. Symmetry and topology together present a certain picture of particle behaviour, provide a classification of particle states, and in some way reveal phenomena such as phase transitions and quantum events.

Sections 2 and 3 provide brief overviews of symmetry and topology, capturing the very essence of these highly abstract disciplines in their contemporary application to particle physics.

Section 4 is devoted to critical views on the existing theories and experimental results in particle physics.

Next, section 5 represents another approach to the same Standard Model, where the mathematical theory of real physical space will be expanded, introducing fundamental concepts such as the primary topological ball, the primary spatial lattice formed by these balls, the physical particle, lepton, quark, as well as concepts such as mass and charge. The presentation will be based on the author's previous works [1–11], the mathematical part of which was carried out jointly with Michel Bounias, who proposed using set theory, topology and fractal geometry to construct physical space. Our theory was verified experimentally in a number of tests conducted in condensed matter physics, biophysics, plasma physics, nuclear physics, and astrophysics (see, e.g. [6, 8, 12–19]). Moreover, one of my technologies, based on the submicroscopic deterministic concept, has been successfully operating in Singapore for 14 years – Alpha Biofuels Pte. Ltd. produces biodiesel on an industrial scale; a special field, called an *inerton field*, is generated in the reactor chamber, which accelerates the chemical reaction of transesterification by about 500 times (see paper [17] for detail).

0.2 Symmetry in particle physics

Introduction of symmetries into particle physics began with Heisenberg's [20] paper, in which he expressed the opinion that the proton and neutron have very similar masses (i.e., they are almost symmetrical), and since it was discovered that the nuclear force is approximately charge-independent, the neutron and proton can be considered as two states of a single entity, namely,

$$p = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad n = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{1}$$

This symmetrical entity was called a nucleon in physical terms, but mathematically it looked like an isospin, that is, a proton and a neutron form an isospin doublet with a total isospin $I = 1/2$ and the third component $I_3 = \pm 1/2$. Then a nucleon state can be written as a superposition described by the formula

$$p = \begin{pmatrix} \psi_p \\ \psi_n \end{pmatrix} = \psi_p p + \psi_n n. \tag{2}$$

Isospin symmetry means that p and n describe an equivalent physics. Such a transformation $N \to N'$ can be described using the matrix U

$$N' = UN \quad \text{where} \quad U = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}. \tag{3}$$

The transformation should not change the physics and hence all probabilities must be the same for N and N' states. The appropriate set of matrices satisfying these conditions forms a special unitary group SU(2).

Continuous transformations fall under a Lie group that is specified with generators of infinitesimal rotations T_k $(k = 1, 2, 3)$, and pass on to transformations along the three independent directions. T_k are half Pauli matrices

$$T_1 = \frac{1}{2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad T_2 = \frac{1}{2}\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad T_3 = \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{4}$$

and T_3 is an operator whose eigenvectors are the p and n states. The following commutations

$$[T_3, T_\pm] = \pm T_\pm, [T_+, T_-] = 2T_3 \quad \text{where } T_\pm = T_1 \pm iT_2. \tag{5}$$

It is interesting that the commutation relations (5) allows one to present the matrix of the T_3 operator acting on T_-, T_3, T_+ as

$$T_3 = \begin{bmatrix} -1 & & \\ & 0 & \\ & & +1 \end{bmatrix} \tag{6}$$

The matrix form (6) in the adjoint representation indicates that its eigenvalues are precisely the electric charges of the three pions, since they belong to this adjoint representation. Then in the adjoint representation, the charge operator is (see, e.g. [21])

$$Q = T_3. \tag{7}$$

By entering an additional quantum number B, which is called the baryon number, such that

$$Q = T_3 + B/2, \tag{8}$$

we obtain the correct charges for protons and neutrons if $B = 1$, and correct charges for pions if $B = 0$.

In a good review pedagogical paper [21] the authors explain in detail how isospin symmetry was extended via a Lie group to the mathematical theory of the group SU(3), which unified subatomic particles (protons, neutrons, and mesons) into an organisational system of hadrons assembled into groups of eight, or octets. Hadrons are classified based on their properties, which mainly consist of charge and a quantum number called "strangeness".

The authors of [21] emphasised that the discovery of the Λ^0 baryon exhibited a something strange about this particle; observations showed that its lifetime is different for decay (10^{-23} s) and production (10^{-10} s), which means that the production and decay were very different mechanisms for Λ^0 (and the same was true for the kaons K).

To insert these strange particles into isospin representations, one needs to put Λ^0 in the singlet state, since no other particle can be found as a SU(2) partner of a Λ^0 baryon. In such a case the expression (8) for the charge operator should change again because it must present the correct charge of Λ^0 and $K^{(0,\pm)}$ [21]

$$Q = T_3 + (B + S)/2 \tag{9}$$

where quantum numbers $S = \pm 1$.

After the discovery of new strange particles (lambdas and kaons), SU(2) symmetry was extended to SU(3) primarily in the context of the Sakata model [22] and then, eventually, to the Eightfold Way, which led to the quark model [23, 24]. Following the same arguments as for SU(2), it was found that the generators of infinitesimal SU(3) transformations must be Hermitian traceless matrices.

Thus, particle theorists became to describe fundamental interactions in particle physics, like the strong force, in terms of gauge transformations. Unlike Abelian theories, like electromagnetism (U(1)), non-Abelian theories use gauge groups where multiplication is not commutative, such as SU(3) for the strong interaction. This non-commutative property leads to self-interactions between force-carrying particles, gluons, and give rise to unique phenomena known as colour confinement.

Nowadays, the use of noncommutative geometry in particle physics has become widespread. It provides a mathematical framework for extending classical geometry to spaces where the coordinate functions do not commute. This framework has been used in theoretical particle physics to develop peculiar abstract models that go beyond the Standard Model by reinterpreting the fundamental forces and particles as arising from the geometry of a noncommutative spacetime. While still highly speculative, these models try to make predictions that could be tested experimentally in high-energy physics and cosmology (even despite the fact that such an approach cannot be tested in any way).

The authors of Ref. [21] explain that similar to a real vector space, the space of all complex 3×3 matrices has dimension 2×3^2. Making it Hermitian decreases the dimension in half, leaving 3^2, but the traceless condition lowers the dimension once more by 1, leaving us with a dimension of $3^2 - 1$. This means SU(3) has 8 linearly independent generators. In SU(2), the usual generators are given by the Pauli matrices. For 3 dimensions, a usual choice of generators, called the Gell-Mann matrices, given by 8 matrices λ_a where $a = \overline{1, 8}$. Among them, $\lambda_{(1,2,3)}$ are simply the Pauli matrices inserted into a larger 3×3 paradigm. Therefore, representations of SU(3) contain representations of SU(2).

Instead of the commutations (5), in SU(3) there are much more other commutations; in addition to the $T_\pm$ operators, ladder operators $U_\pm$ and $V_\pm$ appear.

Rather than taking p, n and Λ (Sakata's [22] basis) as belonging to the fundamental, Gell-Mann [23] and Ne'eman [24] proposed that these should be classified together with the Σ's and Ξ's in the 8 representation. This approach became the famous “Eightfold Way” of particle physics.

The remaining question is [21]: what is the fundamental representation in this case? Which are the fundamental constituent particles of mesons and baryons? Gell-Mann dubbed these particles "quarks," although he did not consider them to be particles, but rather some mathematical characteristics. The three states of the fundamental representation were called up, down and strange quarks, or, in terms of eigenvalues,

$$u = \left|\frac{1}{2},\ \frac{1}{3}\right\rangle, \quad d = \left|-\frac{1}{2},\ \frac{1}{3}\right\rangle, \quad s = \left|0,\ -\frac{2}{3}\right\rangle. \tag{10}$$

Applying the expression (9) to the fundamental representation (10) the researchers [21] derive that these quarks have charges $Q_u = 2/3$, $Q_d = -1/3$, and $Q_s = -1/3$. Their charges are thus fractions of the elementary charge. Now within the Eightfold Way, the fractional baryon number changes to B=1/3, since they are made up of 3 quarks, and mesons are made up of quarks and antiquarks, and therefore have $B = \frac{1}{3} + (-\frac{1}{3}) = 0$.

Thus, the idea of similarity of particles was extended to the quark model, where the "flavour symmetry" suggested that the strong force (i.e. the nuclear force) treats quarks of different masses as nearly identical. For example, u and d quarks have very similar masses, and therefore a symmetry principle predict that they should be produced in equal numbers. At the same time it was recognised that quarks have three different "colours" (due to three fractional charges) and 8 "mediators" (called colour gluons) that provide direct interaction between quarks. This is in short how quarks were theoretically introduced in particle physics in the early 1960s. First experimental evidence of the existence of quarks was conducted in deep inelastic scattering experiments at the Stanford Linear Accelerator Centre (SLAC) in California in 1968.

0.3 Topology in particle physics

Topology was initially applied to quantum field theory, which produced topological quantum field theory.

In classical field theory, a real scalar field $\phi(\mathbf{x}, t)$ means a real number at each point in space, where $\mathbf{x}$ is the position vector and t is the time. In quantum field theory, the real scalar field ϕ is replaced by a quantum field operator $\hat{\phi}(\vec{x}, t)$ that includes the annihilation operator $\hat{a}_{\vec{p}}$ and creation operator $\hat{a}^{\dagger}_{\vec{p}}$ and the angular frequency $\omega_{\vec{p}}$ written for a particular momentum $\mathbf{p}$:

$$\hat{\phi}(\vec{x}, t) = \int \frac{d^3}{(2\pi)^3} \frac{1}{\sqrt{\omega_p}} (\hat{a}_{\vec{p}} e^{-i\omega t + i\vec{p}\cdot\vec{x}} + \hat{a}^{\dagger}_{\vec{p}} e^{i\omega t - i\vec{p}\cdot\vec{x}}) \tag{11}$$

with the commutation relations

$$[\hat{a}_{\vec{p}}, \hat{a}^{\dagger}_{\vec{p}}] = (2\pi)^3\delta(\vec{p} - \vec{q}), \quad [\hat{a}_{\vec{p}}, \hat{a}_{\vec{p}}] = [\hat{a}^{\dagger}_{\vec{p}}, \hat{a}_{\vec{p}}\dagger] = 0 \tag{12}$$

and the vacuum state $|0\rangle$ is defined by $\hat{a}_{\vec{p}}|0\rangle = 0$.

In topological field theories, correlation functions are metric-independent calculations of the correlations between different topological operators. Because they are defined on a spacetime that is independent of its metric, these correlation functions are topological invariants – they do not change under any continuous deformation of the spacetime. This means the results are independent of the shape, size, or location of the manifold, but are instead determined by its topological properties, such as the number of holes, separate parts, or other features. Topological field theories do not apply to flat Minkowski spacetime, as this leads to trivial topological invariants, but they are applicable to curved spacetimes (for example, Riemann surfaces). These theories made their impact by the way they reproduce topological invariants of certain smooth manifolds.

Atiyah-Singer index theory [25, 26] has become a common tool in quantum field theory, which can be applied to topology, as shown in the simple example below [27]: If an action functional depends only on smooth structure of a manifold then corresponding physical quantities (in particular, the partition function) should have the same property, and the simplest example is a functional

$$S = \int_M A \wedge dA \tag{13}$$

where A can be a 1-form on three-dimensional compact manifold M (A may include matter, gauge, ghost and auxiliary fields etc.). This functional is invariant under the gauge transformations $A \to A + d\lambda$ and can be calculated in an explicitly if a gauge condition is imposed on its partition function. The gauge condition cannot be invariant with respect to diffeomorphisms and should involve some additional data, for example Riemannian metric, degenerate quadratic functionals or something else [27].

Atiyah-Segal axioms [25, 26] can further be related to physics, namely, quantum field theories. For example, let Γ indicate the 3D physical space and the extra dimension in $\Gamma \times I$ represent "imaginary" time. The space Z(Γ) is the Hilbert space of the quantum theory. Let the Hamiltonian H characterises a physical system and its time evolution operator is e^{itH}, or an "imaginary" time operator e^{-tH}. The equality $H = 0$ is the main feature of topological quantum field theories since there is no motion of the physical system along the cylinder $\Gamma \times I$ (although for the Lagrangian the inequality $L \neq 0$ may hold). Nevertheless, a tunnelling process from Γ_0 to Γ_1 through a mediate manifold M with $\partial M = \Gamma_0^* \cup \Gamma_1$ reflects the topology of M. ∂M is considered as the vacuum state defined by M. Then the number $Z(M)$ is the vacuum expectation value and in analogy with statistical mechanics it can also be called the partition function.

Quantum Chromodynamics (QCD) is an abstract theory, which particle theorists use for the study of the behaviour of quarks, can be examined in the framework of perturbation theory, and many important results have been obtained within this approach. Topology is completely outside the domain of perturbation theory and, therefore, within a perturbative approach the contribution of the topological term would always be zero [28]. However, topology in QCD is related to the mechanisms of chiral and axial symmetries breaking and restoration; chiral symmetry breaking at low temperatures, and instanton models combined with perturbation theory at very high temperatures, dictate the behaviour of the topological observables in these limiting situations.

The application of topology for QCD was presented in paper [28]. The work examines the connection between the topological properties of QCD, vacuum and the physics of hypothetical axions, including lattice simulations. The authors of Ref. 28 point out that among the most fascinating aspects there is the possibility of including a topological term into the QCD Lagrangian, which leads naturally to the prediction of a yet-unobserved particle – the QCD axion and the same topological term solves an apparent mystery of the hadron spectrum, giving a mass to the η' meson.

As derived by 't Hooft [29], strong interactions of the Standard Model, QCD, possess a non-trivial vacuum structure that in principle permits violation of the combined symmetries of charge conjugation and parity (CP). The QCD Lagrangian admits a charge-parity (CP) violating term

$$L = L_{\mathrm{QCD}} + \theta \frac{g^2}{32\pi^2} F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a \tag{14}$$

where $\frac{g^2}{32\pi^2} F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a$ is known as the topological charge density $q(x)$, and $\theta q(x)$ is called the θ-term. Without the θ-term strong interactions conserve CP. The researchers believe that this term provides an electric dipole momentum d_n in the neutron, which they estimate with QCD sum rules as $d_n \approx 3 \times 10^{-16}\theta$ e cm. The measured value of the neutron electric dipole moment is equal to $\sim 10^{-26}$ e cm, which is leading to the bound $\theta < 0.5 \times 10^{-10}$, and this unnaturally small value of θ has been known since long. This situation is called the strong CP problem.

Hence, the partition function of QCD is a function of θ

$$Z(\theta, T) = \int D[\Phi] e^{-T\Sigma^j \int d^3x L(\theta)} = e^{-VF(\theta,T)} \tag{15}$$

where T is the temperature and $F(\theta, T)$ is the free energy density. $F(\theta, T)$ is related to the probability P_Q of finding configurations with a given topological charge $Q = \int d^4xq(x)$:

$$P_Q = \frac{1}{2\pi}\int_{\pi}^{\pi} d\theta e^{-\theta Q} e^{-VF(\theta)}. \tag{16}$$

The right-hand side of this expression can be expanded into the Taylor series and the coefficients C_n are given by the cumulants of the topological charge [28]

$$C_n = (-1)^{(n+1)} \frac{d^{2n}}{d?^{2n}} F(\theta, T)|_{\theta=0} = \langle Q^{2n}\rangle_{\text{comn}}. \tag{17}$$

The free energy $F(\theta, T)$ as a function of θ reaches a minimum at $\theta = 0$, which dose not solve the strong CP problem and, therefore, this means that the parameter θ has to be considered as dynamic.

So, the axion field $a(x) = f_A\theta(x)$ becomes a space-time dependent parameter θ where the axion decay constant f_A should be much larger than the QCD scale; astrophysical observations give the approximate limits $10^{12} \geq f_A \geq 4 \times 10^8$ GeV [28]. The axion-QCD Lagrangian reads [28]

$$L = L_{\text{QCD}} + \theta\frac{g^2}{32\pi^2} F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a + \partial^2_\mu a^2 + \frac{a}{f_A}\frac{g^2}{32\pi^2} F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a. \tag{18}$$

$F(\theta, T)$ is used to compute the axion mass m_A from the expression

$$m^2_A(T) f^2_A = \frac{\partial^2 F(\theta, T)}{\partial\theta^2}|_{\theta=0} \equiv \chi_{top}(T). \tag{19}$$

The researchers emphasise that there is a close relation between the axion mass and topological susceptibility (19), which is valid for any temperature.

The researchers think that lattice formulation of QCD, which allows calculation of topological and other physical quantities in numeric simulations, is a prospective approach. In Lattice QCD the continuous spacetime is replaced by discrete 4D Euclidean lattice, that is all. Therefore, discretised versions of operators defined on sites of the lattice are introduced. The analysis scheme for a partition function remains practically the same as briefly described above.

In the framework of lattice topology, the topological susceptibility χ_{top} makes it possible to introduce a topological charge density in pure gluonic Yang-Mills theory:

$$q(x) = \frac{g^2}{32\pi^2}\epsilon_{\mu\nu\rho\nu} Tr[F^{\mu\nu}(x)F^{\rho\sigma}(x)] \tag{20}$$

where $F^{\mu\nu}(x)$ is continuous field strength tensor. Then, the topological charge is defined as

$$Q = \int d^4xq(x) = \frac{a^4}{32\pi^2}\epsilon_{\mu\nu\rho\nu}\sum_n Tr[F^{\mu\nu}_{\text{lat}}(n)F^{\rho\sigma}_{\text{lat}}(n)] \tag{21}$$

where a is the constant of lattice QCD and the summation is over all sites of the lattice. The topological charge calculated on the lattice from Eq. (21) is non-integer and requires further renormalisation.

In lattice QCD, topological charge should be an integer value that classifies different vacuum states of the theory based on their “winding number,” which is a property of the gluon field configurations. It is an invariant that measures the non-trivial topology of the gauge field and is crucial for understanding phenomena like chiral symmetry breaking, which relates to the masses of particles like the eta prime meson.

Here are two examples of topological charges. 1) The gluon field configurations around quarks in QCD can have non-trivial topologies meaning they can be deformed into each other continuously. For instance, a bottle, a tea cup, and a doughnut are topological equivalent. Such classes of configurations are distinguished by the topological charge. 2) Instantons are field configurations with a topological charge $Q = \pm 1$, which corresponds to a different winding numbers of the gauge field; these configurations are separated by topological barriers, and the transition between them is a tunnelling effect called instanton.

0.4 Criticism of existing theories in particle physics

To a greater extend, criticism of particle physics theories includes first of all the Standard Model's incompleteness: failing to include gravity, to explain the origin of neutrino masses, explain dark matter and dark energy, or the matter-antimatter asymmetry.

At the same time, the failure of experiments to find predicted particles (e.g., the magnetic monopole, the supersymmetric (SUSY) particles, the sterile neutrino, and the axion) raises serious questions about the validity of the theoretical knowledge base developed by theorists. That is, whether the basic principles and strategy that have guided particle physics for decades are correct.

Besides, the methodology's reliance on aesthetic principles like simplicity and elegance leading to “dead ends”, and a lack of reproducibility and public data transparency. Other concerns involve a focus on finding new particles and justifying high costs for experiments like colliders, which are extremely expensive and therefore cause public dissatisfaction.

0.4.1 QCD and Electroweak interaction are sinking?..

Perhaps the most meticulous critic of the theoretical foundations of the Standard Model is Eliyahu Comay [30–36]. In his works, Comay emphasises that any theory in particle physics must be a relativistically self-consistent quantum theory, but QCD and Electroweak theory do not satisfy this fundamental principle.

Comay [33] reasonable note that particle physicists avoid discussing the experimental data on the photon-electron scattering (Compton scattering); they ignore the effect. This is because the photon total cross sections on neutrons and protons are nearly the same, which points that the photon interaction does not depend primarily on the charge of the nucleon and the same is true with the electron-nucleon deep inelastic scattering, but the Standard Model has no explanation for the hard photon-nucleon interaction [33] (and the same for the electron-nucleon deep inelastic scattering).

The reason must probably lie in the presence of similar magnetic and electrical internal characteristics of both the proton and the neutron. In other words, it is certain that quarks must have both electric and magnetic charges, i.e., they must have the inherent structure of both charge and monopole.

Comay [33] remarks that QCD has no explanation for a significant difference in the electron-proton scattering. In the first case the total cross section decreases monotonically, and the relative portion of the elastic scattering becomes negligible; in the second case, at a high energy proton-proton scattering, the total and elastic cross section begins to rise, and the relative portion of the elastic cross section takes a uniform value of about 1/6.

One more Comay's remark [33]: "QCD cannot explain the increase of the cross section of the high energy proton-proton data. The interpretation of this effect is ignored by QCD textbooks, and this negligence substantiates the previous assertion." Other kinds of QCD discrepancies are also discussed by Comay [33].

Comay's [32, 34, 36] analysis proves that the electroweak theory of $W^{\pm}$ particles has no coherent expression for the 4-current of these particles, and Z particle does no obey the de Broglie principle of wave-particle. Hence, he reasonably states that these failures justify the rejection of the electroweak theory. Comay [35] highlights that the Standard Model electroweak theory goes against the data: "It ignores the similarity between strong and electromagnetic interactions, while it unifies the weak and the electromagnetic interactions that have contradictory experimental attributes. This issue casts doubt concerning the fit between the electroweak theory and the real world." And further: "The electroweak

Lagrangian density (5) is wrong because the previous points prove that its m^2 term violates SR (special relativity)."

Comay [35, 30] also notes that the electroweak theory of the Z particle and the theory of the Higgs boson H are wrong because in line with these theories the named particles are elementary particles that are described by a mathematically real quantum function. However, such a function cannot describe the wave attributes of a massive quantum function. Furthermore, theories of a mathematically real quantum particle violate the Weinberg correspondence principle because quantum mechanics uses mathematically complex functions [35].

Finally Comay [35] remarks: "It is impossible to write a coherent expression for the electromagnetic interaction of the electroweak's description of the charged particles $W^{\pm}$. Although Maxwellian electrodynamics is a generally accepted theory, electroweak textbooks strangely ignore this $W^{\pm}$ inherent fault."

In my works [7, 8, 10] it is argued that quarks have only integer charges, $\pm e$, and they can also be in a magnetic monopole state g. The outcome obtained have never been examined experimentally taking into account the possible presence of magnetic monopoles in hadrons. This tunnel vision when examining the data may indicate quite serious errors in the interpretation of the results. All this automatically classifies QCD theory as a fringe theory.

It is known that a positron moves through matter without immediate annihilation, although theory predicts instantaneous annihilation [37]; furthermore, if the positron were identical to the electron, only having the opposite sign of charge, the positron's trajectory should be symmetrical, like a mirror image of the electron's trajectory, but it is not! The process of particle-antiparticle generation practically omits the electrical interaction between positron and electron, but why the energy threshold is lowered?..

An abstract discipline called noncommutative geometry has entered the description of fundamental physics. It offers a new abstract basis for unifying and rethinking fundamental physical theories, considering the forces of the Standard Model and gravity within a single noncommutative geometric structure, in which coordinates do not commute. There is also string theory, which also seeks a unified description of all known particles and interactions within the framework of complete abstractionism. These two areas of abstract research do not involve comparison with experiments at all.

0.4.2 More than 100 years ago...

Thomson scattering is a scattering at which a free charged particle elastically scatters electromagnetic radiation. For the electron, Thomson [38] found that the scattering cross-section is $\sigma = 8\pi r_e^2/3$ where he introduced a parameter known as the classical electron radius $r_e = 2.8179403227 \times 10^{-15}$ m.

Compton [39] examining quantum mechanical properties of a particle, namely, the scattering of X-ray photons by electrons discovered a scattering length $\lambda_{\mathrm{Com}} = h/(mc)$, which is known now as the Compton wavelength; for the electron it is $\lambda_{\mathrm{Com}} = 2.4263102367 \times 10^{-12}$ m.

These two experimental results directly demonstrates that both the classical electron radius r_e and the Compton wavelength λ_{Com} are real characteristics of the electron (as well as any other particle). However, neither the Compton wavelength, nor the classical electron radius is included in any modern theory of particle physics. Strange as it may sound, these two important characteristics found more than 100 years ago remained outside the framework of the theories on which the Standard Model is based.

Thus, particle physics does not have a solid theoretical foundation, it is not based on physical laws and uses only abstract mathematical models that in practice do not provide answers to specific questions, primarily regarding the physical processes of particle creation, interaction and decay. Particle physicists for more than a century could not understand what ultimately happens in a simple cloud chamber, but went further to large colliders, where they produced even more large dark spots.

0.4.3 Quantum mechanics is bursting at the seams?..

Not only does particle physics have internal problems, quantum mechanics is not so simple either; it is also very problematic. Shpenkov and Kreidik [40–47] examined the application of the Schrödinger equation to the hydrogen atom problem. They demonstrated that in a spherical coordinate system, the solution for the radial variable, written by Schrödinger as a series, was arbitrarily truncated without mathematical rigour. Of course, this move led to obtaining discrete levels of the energy of the hydrogen atom

$$W = \frac{Ze^2}{8\pi\epsilon_0 a_{\mathrm{B}} \rho_{nlj}} \tag{22}$$

where a_{B} is the Bohr radius. The radii of shells of the most probable states in the atom are $r = a_0 \rho_{nlj}$ where j is the number of the roots of the extremum. However, these roots are not equal to the integers squared, i.e., $\rho_{nlj} \neq q^2$ (where $q = 1, 2, 3, ?$), and, therefore, Schrödinger's spectra (22) do not coincide with real spectra of the hydrogen atom, i.e. the Lyman series, Balmer series, Pashen series, Brackett series and Pfund series. Although textbooks praise Schrödinger's success in solving the equation for the hydrogen atom, the actual result is the opposite – Schrödinger did not calculate the correct discrete spectra. Moreover, he was not allowed to truncate the series, but did so intentionally, because otherwise he would have obtained a continuous spectrum.

Shpenkov and Kreidik [40–47] further showed that the correct spectral series of the hydrogen atom can be obtained if one uses not Schrödinger's equation, but a classical wave equation

$$\Delta\psi(\vec{r}, t) - \frac{1}{\upsilon^2}\frac{\partial^2}{\partial t^2}\psi(\vec{r}, t) = 0. \tag{23}$$

The solutions for the radial variable are provided by spherical and cylindrical Bessel functions, which are well known in wave field theory. The polar-azimuth functions give the wave numbers l and m. The orbital electron in the hydrogen atom is described by the cylindrical wave field. The spectra of Balmer series and the others consent well with Shpenkov-Kreidik's theory. They check their theory also on practically all known atoms.

Why does the classical wave equation work, but the Schrödinger equation falters? The point is that Schrödinger [48] used the classical wave equation to derive his own. Looking for a solution to equation (23) in the form of a plane wave $\exp[ir/(2\pi\lambda) - i\nu t/(2\pi)]$ he used the Louis de Broglie [49] relationships

$$E = h\nu, \quad \lambda = h/p \tag{24}$$

and obtained the equation

$$\Delta\psi(\vec{r}, t) + \frac{8\pi^2 m}{h^2}(E - V)\psi(\vec{r}, t) = 0. \tag{25}$$

Then Schrödinger and other physicists began to consider equation (25) as independent of the wave equation (23). The solution of spectral problems using the stationary Schrödinger equation was always successful. But Shpenkov and Kreidik saw a certain catch in this.

Thus, it turns out that the Schrödinger equation is also not fundamental.

So, it is better to use a usual wave equation. The wave equation must propagate in some real substrate, and therefore this means that all the foundations of physics need a deep revision.

0.4.4 Shadows of the past?..

As is known, at the beginning of the 20th century, Hendrik Lorentz and Henri Poincaré developed the theory of kinematic transformations of a moving body and the perception of ether as a substance that exists did not prevent them from obtaining the corresponding formulas. Albert Einstein's 1905 paper on special relativity gave impetus to the rejection of the ether. However, both in the presence of ether (according to Lorentz and Poincare) and in its absence (according to Einstein), the kinematic formulas of transformations look the same.

Different researchers have put different mechanical meanings into the behaviour of the ether, but the fixation of the corresponding phenomena or effects has not yielded reliable results. It seems that due to the inability to capture any mechanical properties attributed to the ether, it was rejected in favour of a physical vacuum, which is something incomprehensible, but endowed with quantum properties.

Nevertheless, it is necessary to note one interesting article on electron dynamics by Poincaré [50] of 1908; he wrote that the electron is moving, surrounded by excitations of the ether. This is a very interesting remark that indicates Poincaré's deep understanding of the nuances of physics.

But how did the concept of ether, as a thin invisible matter that permeates the entire universe, come about? In his influential cosmological work *De caelo* (*On the Heavens*), the ancient Greek philosopher Aristotle theorised the existence of aether (or ether), a divine, unchanging fifth element that filled the universe beyond Earth's atmosphere. Before that, Anaximander, an ancient Greek philosopher, proposed a concept of apeiron that is similar in many respects Aristotle's ether [51]. The apeiron was treated as the primary source from which everything originates. It was not a material substance, but an indefinite entity that predates any specific quality. All qualities emerge from the apeiron leading to the formation of the visible world, the apeiron acts as a cosmic law maintaining eternal cosmic equilibrium. All things are temporary forms, they emerge from the apeiron and eventually return to it upon their destruction. This creates an eternal cycle of birth and destruction, where the universe is self-regulating and self-renewing.

The theory of ancient Greek scholars Leucippus and Democritus [52] centred on the idea that the universe was composed solely of atoms and the void; the weight of atoms (i.e., mass) vary with their size. They did not include the concept of ether in their model of the cosmos. Leucippus and Democritus's atom means an indivisible particle; from such particles the cosmos should be composed. Each atom may have concavities and convexities. Atoms can have different sizes and shapes, they can contact each other, clinging like hooks, and can vibrate.

It is known that Democritus visited Egypt, Persia and possible India. He studied with Chaldean and Magi priests. The Magi (Maga-Brahmans) were a group of priests in ancient Persia (including a southern part of modern Ukraine) and India, and they were associated with the introduction of sun-worship [53]. The Magi were undoubtedly the guardians of the Vedic heritage. Roy [54], as a physicist, reading ancient Vedic books, saw in the texts encoded knowledge of physical space, the universe and elementary particles. He informed that real space was called in Sanskrit "loka": loka has a web-like structure, it is composed of indivisible cells; cells are characterised by their interface and each cell represents a subtle particle; the electric charge is kept and plays on the surface of a particle. "Inspite of the material body being subject to destruction, the subtle particle is eternal" (*Bhagavat-Gita*, 2.18) [11].

Thanks to modern advanced technologies and mobile communications, we can now watch many places on the planet where modern and ultra-modern technologies were used at least 10,000–15,000 years ago. This indicates that we are not the first civilisation on Earth. Then the Vedic heritage is nothing other than a remnant of knowledge from our predecessors, whose civilisation perished due to reasons unknown to us. But in this case we must admit that there is no undefined ether and no vacuum (emptiness), but there is space, loka, endowed with a network, or lattice structure in which each cell (Democritus' atom) is provided with special natural properties – size, shape and interface.

We know that elementary particles can be born at any point on our planet, which means that it is the same in the entire Universe. Then it is logical to assume that particles can only be born from cells of space. That is, when the configuration of two neighbouring cells changes spontaneously or at some peculiar conditions, two particles with opposite charges should be born from the cells.

Thus, a mathematical theory of real space is clearly needed, which, in turn, must become the correct generator of the fundamental laws of physics.

0.5 Geometry and topology of real space

0.5.1 Distance, measure, founding element and founding lattice

So, real space requires a reliable mathematical cell theory. Such theory in fact was developed by Michel Bounias and the author [1–3], see also [4, 8, 11]. The main characteristic of such a space should be based to the fact that distances and matter (particles) must arise from the same manifold.

First, we analysed the concepts of measure and distance in a broad topological sense, in particular for estimating the dimensionality of a space, since sets or spaces and functions measurable under different conditions are usually interconnected.

A measure is treated as the measured object with some unit taken as a standard. A set measure on E is a mapping m of a B of sets of E in the interval $[0, \infty]$, exhibiting denumerable additivity for any sequence of disjoint subsets (b_n) of B, and denumerable finiteness, i.e., the following correspondence must be fulfilled:

$$\mathsf{m}(\cup_{n=0}^{\infty} b_n) = \cup_{n=0}^{\infty} \mathsf{m}(b_n) \tag{26}$$

where $\exists\, b_n$, $b_n \in B$, $E = \cup b_n$, $\forall \in \mathrm{N}$, $\mathsf{m}(b_n)$are finite.

The unit of measurement (standard) is the part subject to a gauge (J). Calibration is a function defined on all bounded sets of the considered space and it has the properties: 1) a singleton has measure naught: $\forall x, J(x) = 0$; 2) (J) is continued with respect to the Hausdorff distance; 3) (J) is growing: $E \subset F \Rightarrow J(E) \subset J(F)$; 4) (J) is linear: $F(r \cdot E) : r \cdot J(E)$.

So, we have a defined concept of distance in topology: diameter/size or deviation; such distance can be applied to fully ordered sets.

The Jordan and Lebesgue measures demand respective mappings (I) and (m^*) onto spaces with operators $\cap, \cup$ and $\complement$. In spaces of the $\mathbb{R}^n$ type, the ball tessellation must also be involved. This means that a distance must be available for the measure of diameters of intervals.

Since the set of measure zero is a linear set (E), all its points lie in intervals whose sum is less than (e).

Regarding distance. The path $\phi(A, B)$ is a set composed as $\phi(A, B) = \cup_{a \in A, b \in B} \phi(A, B)$ and they are defined in a sequence interval $[0, f^n(x)]$, $x \in E$. The relative distance of A and B

in E, noted $\Lambda_E(A,B)$ is contained in $\phi(A,B)$:

$$\Lambda_E(A,B) \subseteq \phi(A,B). \tag{27}$$

In an ordered space, the distance d between A and B is

$$d(A,B) \subseteq \mathrm{dist}(\mathrm{inf}A, \mathrm{inf}B) \cap \mathrm{dist}(\mathrm{sup}A, \mathrm{sup}B), \tag{28}$$

and the distance is evaluated through either classical forms or even the set-distance $\Delta(A,B)$.

The set-distance is the symmetric difference between sets and that it can be extended to manifolds of sets and it possesses all properties of a true distance. In a topologically closed space, such distances are the open complementary of closed intersections called "instances" by Bounias. The intersection of closed sets is closed and the intersection of sets with nonequal dimensions is also closed. Hence the instances stands for closed structures. This reflects physical-like properties, i.e. they characterise objects, and the distances as being their complementaries constitute the alternative class. Thus, a physical-like topological space tends globally to be subdivided into objects and distances as full components.

Any topological space is metrisable as provided with the set-distance Δ, which is a kind of a metric space called the "delta-metric space", and it is a natural metric. A distance $\Delta(A,B)$ is a kind of an intrinsic case $[\Lambda_{(A,B)}(A,B)]$ of $\Lambda_E(A,B)$ while $\Lambda_E(A,B)$ is a "separating distance". The separating distance also stands for a topological metrics. Hence, if a physical space is a topological space, it will always be measurable.

Space dimensions play a very important role in understanding the structure and properties of physical space. If the structure of members of a set is unknown, a problem arises how to distinguish unordered N-tuples and ordered N-tuples. The problem is crucial for the assessment of the actual dimension of a space.

Let a fundamental segment (AB) has intervals $L_j = [A_j, A_{(j+1)}]$, a generator is composed of the union of several such intervals $G = \cup_{(j\in[1,n])} L_j$ and the similarity coefficients be defined for each interval by $\varrho_j = \mathrm{dist}[A_i,\, A_{(j+1)}/\mathrm{dist}(AB)] < 1$. The similarity exponent of Bouligand, e, is such that for a generator with n parts

$$\Sigma_{j\in[1,n]}(\varrho_j)^{\mathrm{e}} = 1. \tag{29}$$

When all intervals are almost the same size, the various dimension approaches are indicated in the resulting relations

$$n \cdot (\varrho_j)^{\mathrm{e}} = 1, \quad \mathrm{e} \approx -\mathrm{Log}\, n/\mathrm{Log}\, \varrho > 1. \tag{30}$$

When e is an integer, it displays a topological dimension showing that a fundamental space E can be tessellated with an entire number of identical balls B exhibiting a similarity with E, upon coefficient ϱ.

In a space composed of members identified with some abstract components, it may not be found tessellating balls all having identical diameter. Then the measure should then be used as a probe to estimate the aspect ratio ϱ needed to calculate the dimension.

Let a space X be a $\mathtt{N}$-object where $\mathtt{N}$ is the number of vertices, i.e. members in X, $k = (d - 1 = N - 2)$. X can be decomposed into the union of balls represented by D-faces A^{D} where A is the distance. This D-face is a D-simplex S_j whose size, as a ball, is evaluated by S_j^{D}. Let N be the number of such balls that can be filled in a space H, so that

$$\cup_{j=1}^{N} S_j^{\mathrm{D}} \subseteq (H \approx L_{\max}^{\mathrm{d}}) \tag{31}$$

where H is the ball whose size is evaluated by L^{d}, L is the size of a 1-face of H, and d is the dimension of H. If $\forall S_j$, $S_j \approx S_0$, then the dimension of H is

$$\mathrm{Dim}(H) \approx -(\mathrm{D} \cdot \mathrm{Log} S_0 + \mathrm{Log} N)/\mathrm{Log} L_{\max}^{1} > 1. \tag{32}$$

The relation (32) stands for an interior measure in the Jordan's sense. In contrast, if we assume that the reunion of balls covers the space H, then $\mathrm{Dim}(H)$ will rather represents the capacity dimension, which remains an evaluation of a fractal property.

The existence of the empty set $\emptyset$ is a necessary and sufficient condition for the existence of abstract mathematical spaces (W^n) endowed with topological dimensions n. The empty set is depicted as a set without elements and simultaneously containing empty parts. This means that the empty set represents self-similarity at all scales, as well as derivability from nothingness. These two characteristics designate fractal structures. In essence, the empty set $\emptyset$ becomes a founding element.

Providing the empty set $\emptyset$ with operators $\in$ and $\subset$, and the combination rules that the complementarity $\complement$ dispenses leads to a definition of magma that allows a consistent application of De Morgan's first law without violating the axiom of foundation if and only if the empty set is considered as a hyperset, which is a not well-founded set. This gives on to

Bounias' Theorem [1]: The magma $\emptyset^{\emptyset} = \{\emptyset, \complement\}$ constructed with the empty hyperset and the axiom of availability is a fractal lattice.

Remark. A magma is a set equipped with a single binary operation that must be closed by definition. Writing $\emptyset^{\emptyset}$ denotes that the magma reflects the set of all self-mappings of $\emptyset$, which emphasises the forthcoming results.

The proof of this theorem [1] clearly shows that the mathematical lattice of primary topological balls is a fractal lattice, which was called the tessellattice by Bounias. In application to real physical space this means that the properties of this substrate are characterised by the mathematical laws of the tessellattice. It is reasonable to associate the size of a degenerate cell in the *tessellattice* with Planck's length $\ell_{\mathrm{P}} = \sqrt{\hbar G/c^3} \simeq 1.616 \times 10^{-35}$ m.

The mappings of the Poincaré section S_j into the section S_{j+1} imposes the conservation of the topologies of the general structure of the mapped spaces, which allows changes in the position of objects located inside these structures. Two Poincaré sections, which are mapped, form a set-distance $\Delta(A, B, C, ...)$ and it can be treated as the generalised set distance, or the extended symmetric difference of a family of closed spaces:

$$\Delta(A_j)_{j \in N} = \complement_{\cup\{A_j\}} \cup_{j \neq k} (A_j \cap A_k). \tag{33}$$

The complementary of Δ, which is $\cup_{j \neq k}(A_j \cap A_k)$, in a closed space is closed even if it involves open components with nonequal dimensions. In such a system the state of objects in a timeless Poincaré section can be named the instance, $\mathsf{m} \cup_{j \neq k} (A_j \cap A_k)$. Since distances Δ are the complementaries of objects, the system stands as a manifold of open and closed subparts. Mappings of these manifolds from one section to another, which preserve the topology, represent a reference system in which the topology allows one to characterise possible changes in the configuration of some components. If there are morphisms in the system, then this is a manifestation of a phenomenon similar to motion, when comparing the state of the section with the state of the mapped section.

The spaces referred above can exist upon acceptance of the existence of the empty set as a primary axiom [2].

The composition of the topological distances $\Delta(A, B, ...) = \complement_{A \cup B} ...(A \cap B \cap ...)$ or the topological instances $\mathsf{m}\langle(A, B, C, ...\rangle = (A \cap B) \cup (A \cap C) \cup (B \cap C)...$ with a function f, which indicates the changes occurring in the situation of objects, acting over the populations of objects in the considered sets, leads to a momentum-like structure (MJ) and accounting for elements of the differential geometry of space.

The (MJ), which maps an instance (a 3D section of the embedding 4-space) onto the next, applies to both the open (distances) and their complementaries that are closed (objects) in embedding spaces. Therefore, points corresponding to physical objects can also be contained in both reference structures and two kinds of mappings are formed with each other.

0.5.2 Spacetime as a nonlinear convolution, particles in the tessellattice, and quanta of fractality

Bounias' Theorem [2]. A space-time-like sequence of Poincaré sections is a nonlinear convolution of morphisms. The proof exhibits that the generalised convolution, which is a nonlinear and multidimensional form of the convolution product, exhibits a great similarity with a distribution of functions

$$\langle f, \phi \rangle = \sum \phi(x) f(x), \tag{34}$$

or a convolution product

$$\int f(X-u)F(u)du = (f * F)(X) \tag{35}$$

The connection from the abstract universe of mathematical spaces and the physical universe of our observable ordinary spacetime, i.e. the fundamental metric, is provided by a convolution of morphisms

$$\mathfrak{D}4 = \int \left(\int_{dS_0}^{dS_{\max}} (d\vec{x} \cdot d\vec{y} \cdot d\vec{z}) * d\Psi(x) \right) \tag{36}$$

where dS is the element of spacetime and $d\Psi(x)$ is the function accounting for the extension of 3D coordinates to the 4th dimension through convolution $(*)$ with the volume of space. In such a way, spaces of topologically closed parts account for the interaction and perception, and they meet the properties of physical spaces.

Thus, the convolution (36) of spatial elements determines the appearance of matter, that is, particles, in the universe.

Let we have a lattice $F(U) = \{\cup(\sum_n^n)\} \cup \varpi$ where ϖ is the set with neither members nor parts, i.e. the "nothingness singleton". The ϖ is contained in none of existing sets and it supports both relativistic space and quantic void because 1) the concept of distance and the concept of time have been defined on it, and 2) this space holds for a quantum void since on one hand, it provides a discrete topology, with quantum scales, and on the other hand it contains no "solid" objects that would be associated with physical matter.

Continuity in the perception of spacetime is ensured if the reference frames are preserved by means of homeomorphic mappings. This means that there is no need for exact replication: just topological structures should be conserved, and the realisation of implementation of varieties is allowed even in a space of different dimensions.

Then we can make a **proposition**: The sequence of mappings of one reference structure into another (for example, a couple of elementary cells) represents a fluctuation of the volume of any cell along the arrow of physical time.

But homeomorphic fluctuations can be broken if the mapping follows a certain rule, namely, the transformation of the cell may involve some repeated internal similarity. Then, if N similar figures with similarity ratios $1/r$ are obtained, the Bouligand exponent (e) is given by the relation (30), $N(1/r)^{\mathrm{e}} = 1$, and the image cell gets a dimensional change from d to $d' = -\ln N/\ln r = \mathrm{e} > 1$. Figure 1 clarifies the above. Fractals are geometric shapes that display self-similarity at increasingly smaller scales.

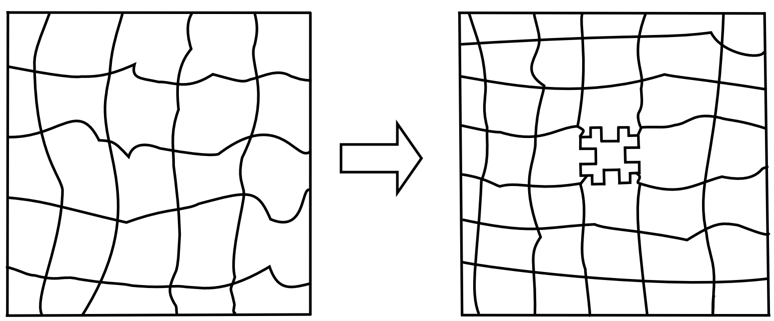

Figura 1: Fractal transformation of the cell generates the appearance of a particle.

The concept of self-similarity, where an object is composed of smaller, scaled-down copies of itself, is a cornerstone of fractal geometry and appears in many areas of mathematics from recursive sequences to certain structures in group theory. Many fractals are created by repeating a simple process, known as iteration, over and over.

So in the tessellattice, the homeomorphic part of the image cell is no longer a continuous figure, and the transformed cell no longer has the property of a reference cell. This fractal transformation means the formation of a "particle", which is also called a "particle cell" or, more appropriately, a "particled ball", since it is a kind of the topological ball $B[\emptyset, r(\emptyset)]$. Thus, the particled ball is represented by a semi-similar transformation in the continuous deformation of the unit cells of space.

How does a particled ball interact with the surrounding degenerate cells of a tessellattice? A minimum fractal structure is provided by a self-similar figure whose combination rule includes an initiator and generator for which the similarity dimension exponent is higher than unity.

Initiator. Due to self-similarity of $\geq \emptyset$, the complementary of itself in itself, the one obtains: $\emptyset \longmapsto \{(\emptyset), \emptyset\}$. That is, one ball gives two identical balls. This is continued into a sequence of $\{\frac{1}{2}, \frac{1}{4}, ...\frac{1}{2n}\}$ numbers at the nth iteration; the initiator providing the needed iteration process is the series $(I) = \sum_{j=1\to\infty}\{(1/2)^j\}$. The terms of (I) are indexed on the set of natural numbers, and thus supply an infinitely countable number of members. In addition, 2^n also denotes the number of parts from a set of n members.

Generator. Let an initial figure (A) be subdivided into r subfigures at the first iteration and hence the similarity ratio is $\varrho = 1/r$. Let $N = (r + a)$ be the number of subfigures constructed on the original one. Then one has $\mathrm{e} = -\mathrm{Ln}(r + a)/\mathrm{Ln}N$ and in this expression the value of the Boulingand exponent e is bounded by unity if r is extended to infinity. For any finite r (which is presumably the case in the physical world), the exponent e is greater than unity. Then

$$\{\min(\mathrm{e})|\,\mathrm{e} > 1\} = \mathrm{Ln}(\max(r) + 1)/\mathrm{Ln}(\max(r)), \tag{37}$$

which completes the description of a quantum of fractality.

Let a fractal system Γ be constructed as $\Gamma = \{(\emptyset), (r+a)\}$. At the nth iteration, the number of additional subfigures is $N_n = (r + a)^n$ and the similarity ratio is $\varrho_n = 1/r^n$. At the jth iteration, subvolume (v_j) is created and in the simplest case $v_j = v_{(j-1)} \cdot (1/r)^3$. Since the number $N_j = (r + a)^j$ of such subvolumes is created at the jth iteration, the total volume covered by the subvolumes formed by the fractal iterations to infinity is the sum of the series

$$v_f = \sum_{j=1\to\infty} \{(r + a)^j + v_{j-1}(1/r)^3, \tag{38}$$

which can be presented as

$$v_f = \sum_{j=1\to\infty} \left\{ \left[\prod_{j=1\to n} (r + a)^{j-1} \right] \cdot (1/r)^3 \right\}. \tag{39}$$

Therefore, fractal decomposition consists in the distribution of members of the set of fractal subfigures

$$\Gamma \supset \left\{ \sum_{j=1\to\infty} \{(r + a)^j \cdot v_{j-1} \cdot (1/r)^3\} \right\} \tag{40}$$

constructed on one figure among a number of connected figures $(C_1, C_2, ..., C_k)$ similar to the initial figure (A). If k reaches infinity, then all subfigures of (A) are distributed and (A) is no longer a fractal.

That is, a ball with its set of fractals can distribute these fractals all around to the nearest balls, such that the ball will lose its fractal dimension though preserving the volume (Figure 2).

However, if it is a particled ball (a ball in which the particle was born), then in this fractal-free state the ball must be rigid due to the acquired tension, as it is to be distinguished from the degenerate cells of the tessellation lattice.

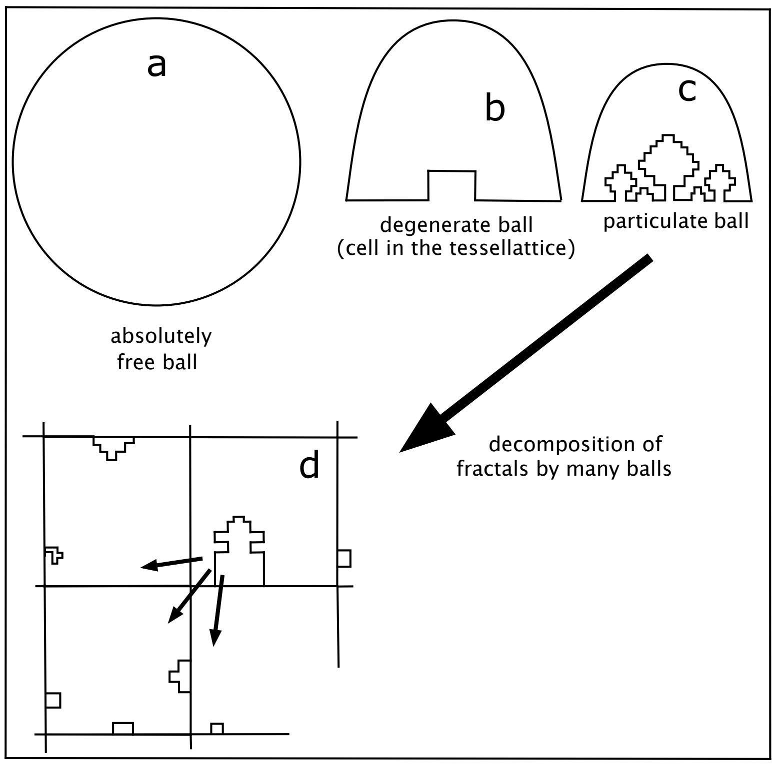

Figura 2: Possible distribution of volumetric fractals from the particulate ball.

Such a picture should arise during the motion of the particled ball, which squeezes between surrounding degenerate balls of the tessellattice and experiences a kind of friction. Fractal decomposition leads to the distribution of coefficients $f(\mathrm{e}_k)$, whose most ordered form is a sequence of decreasing values:

$$f(\mathrm{e}_k)\{(\mathrm{e}_1)_{(j\in]k,1])}\}. \tag{41}$$

From the relation (41), it follows that the remaining of fractality decreases from the kernel (i.e. the zone adjacent to the original particled deformation) to the edge of the cloud of scattered fractals. At the edge, it can be conjectured that, depending on the local resistance of the tessellattice, the last decomposition (denoted as the nth iteration) can result in $(\mathrm{e}_n) = 1$. Thus, while central fractals exhibit decreasing higher boundaries, edge fractals are bounded by a rupture of the remaining fractality.

0.5.3 The concept of lepton and quark, mass and charge

A particled ball, as described above, provides a formalism that describes elementary particles. We know two classes of elementary particles – leptons and quarks. What is the fundamental difference between these two classes of particles? A lepton is a stable particle, but a free quark cannot exist alone. Therefore, we can assume that a lepton is characterised by a contracted fractal state of the corresponding particled ball, while a quark may have an inflated fractal sate. In other words, in the tessellattice a lepton looks like a speck, while a quark looks like a bubble.

Now we need to define the physical notion of mass.

In this respect, mass is represented by a fractal reduction in the volume of the ball, while a simple reduction in volume, as in degenerate cells, which is not obey any law, is not enough to provide mass. Accordingly, if v_o is the volume of an absolutely free cell, then the reduction in volume as a result of fractal concavity will be as follows: $\mathcal{V}^{\text{particle}} = v_\mathrm{o} - v_f$, that is, according to the relation (39):

$$\mathcal{V}^{\text{particle}} = v_\mathrm{o} \cdot \left(1 - \sum_{j=1\to\infty} \left\{ [\prod_{j=1\to} (r+a)^{j-1}]/(r)^{3j} \right\} \right), \tag{42}$$

that is, since $(r+a) = (r)^\mathrm{e}$, we have instead of (42)

$$\mathcal{V}^{\text{particle}} = v_\mathrm{o} \cdot \left(1 - \sum_{(v)} \left(\sum_{j=1\to\infty} \left\{ [\prod_{j=1\to n} (r_v)^{\mathrm{e}_v(j-1)}]/(r)^{3j} \right\} \right)_v \right) \tag{43}$$

where (v) denotes several possible fractal volumetric concavities affecting the particled ball. The relationship (43) relates the volume of particled balls to the fractal dimensional change (e), which can be expressed as the following: The mass m of a particled ball B is a function of the fractal-related decrease of the volume of the ball:

$$m \propto \left(\frac{\mathcal{V}^{\text{degen.}}}{\mathcal{V}^{\text{particle}}} \right) \cdot (\mathrm{e}_v - 1)_{\mathrm{e}_v - 1} > 1 \tag{44}$$

where $\mathcal{V}^{\text{degen.}}$ is the volume of a degenerate topological ball, $\mathcal{V}^{\text{particle}}$ is the volume of the particled ball, (e) is the Bouligand exponent and (e − 1) is the gain in dimensionality given by the fractal iteration as ascribed to the volumetric changes of the ball. If we multiply expression (44) by the dimensional factor C, we will have the physical definition of the mass of the particle.

Just a volume decrease is not sufficient for providing a ball with mass, since a dimensional increase is a necessary condition. A ball contracted in the described way becomes a lepton particle, namely: an electron, muon, tau (and the same for antileptons).

In the case of a quark, the quark mass can be determined from the ratio of the reciprocal to the specified lepton mass (44). Namely,

$$m_{\text{quark}} \propto \left(\frac{\mathcal{V}^{\text{particle}}}{\mathcal{V}^{\text{degen.}}} \right) \cdot (\mathrm{e}_v - 1)_{\mathrm{e}_v - 1} > 1 \tag{45}$$

So, the emergence of a local fractal deformation in the degenerate tessellattice means the appearance of matter.

The aforementioned mathematical small volume fractals in physics are spatial excitations called *inertons*. Inertons carry fragments of mass and are present everywhere in the universe. They also are a substructure of the matter waves, i.e. they fill the so-called particle's wave ψ-function. The inerton field (massotension field) is as fundamental in the Universe as the photon field (electromagnetic field).

The greater the degree of fractal deformation, the greater the mass of the lepton. We know about three leptons, so there are three stable states in a fractally contracted particled ball, and the smaller the radius of the particle ball, the greater the mass. Hence for the lepton family: $\ell_{\text{P}} > \mathtt{r}_{\text{electron}} > \mathtt{r}_{\text{muon}} > \mathtt{r}_{\text{tau}}$.

Unlike leptons, quarks have increasing mass with increasing radius. So, for a family of quarks: $\ell_{\text{P}} < \mathtt{R}_{\text{up}} < \mathtt{R}_{\text{down}} < \mathtt{R}_{\text{strange}} < \mathtt{R}_{\text{charm}} < \mathtt{R}_{\text{bottom}} < \mathtt{R}_{\text{top}}$. Since quarks are bubbles in the tessellattice, the appropriate theory that describes them should be called *quantum bubble dynamics* (QBD).

Now we have to discuss the concept of charge. But first, a general idea – when a local deformation is created in one lattice cell, it should induce smaller, similar fractal deformations in the surrounding cells, which will extend to a certain distance, that is, a radius. Thus, the born particle with mass m is pulled by its volumetric fractal deformation coat to a distance R_m.

A cell in the tessellattice is specified not only by its volume, but also by its interface. So, a quantum of surface fractality can be generated on the surfaced of the created particle. This surface quantum fractal directly indicates that this is the formation of an electric charge. When the quantum of surface fractal deformations collapses into one single ball, two adjacent balls exhibit opposite forms: one in the sense of convexity and the other in the sense of concavity. That is, the born charged particle induces an electric polarization on the neighbouring balls, which is extended all the way to the remote threshold ball (i.e., the cell of the tessellattice) located at a distance R_e.

Since there are two opposite charges in Nature, this means that on a particle, the quantum surface fractal can be oriented either outward (positive charge $+e$) or inward (negative charge $-e$). Figure 3 depicts the charged particle in the tessellattice.

Thus, we see how two opposing conceptual features are formed from a topological ball of the tessellattice: the volume can fractally either decrease or increase, and this characterises the appearance of mass, that is, matter – a lepton or a quark, respectively; the surface can be covered with a fractal quantum oriented inward or outward, and this makes the particle charge negative or positive, respectively.

The motion of mass and charge in the tessellattice has its own peculiarities and belongs to physics, and is described, in particular, in my works [5–10].

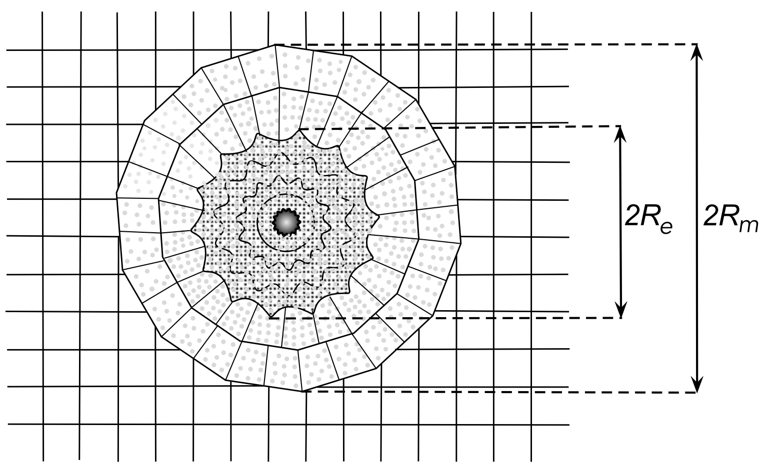

Figura 3: Deformation coat around the created particle in the tessellattice includes two substructures: massive with the radius R_m and electrical with the radius R_e. In the case of an electron, the radius R_m coincides with the electron's Compton wavelength $\lambda_{\text{Com}} = 2.4263102367 \times 10^{-12}$ m, and the radius R_e coincides with Thomson's classical electron radius $r_e = 2.8179403227 \times 10^{-15}$ m. These two electron coats are responsible for the fine structure constant, namely, $\alpha = 2\pi r_e/\lambda_{\text{Com}} \approx 1/137$. This figure is similar for both leptons and quarks.

Finally, this subsection results in the following amendments to the Standard Model. Quarks have only integer charge values $\pm e$ and hence there are no colour flavours in particle physics. Leptons and quarks in motion due to the interaction with the tessellattice periodically transition from a charge state e to a magnetic monopole state g. A particle can be "frozen" in a magnetic monopole state and this is the case for one of the two quarks in pions $\pi^{\pm}$ and one of the three quarks in a neutron, and the neutrino is also a magnetic monopole [10, 7, 8].

These innovations, of course, destroy the entire abstract structure of QCD. In addition, the presence of the tessellattice cancels the electroweak interaction, since the combined particles Z and $W^{\pm}$ are only debris in the particle transformation reactions. A pair of weakly bound quarks, which form these particles, under the influence of pressure from the tessellattice, transitions to two independent leptons [7, 8, 10].

Interaction between particles is provided only by inerton and photon fields, although there is an inverse inerton field between quarks (it can be called the quark-inerton field, or qinerton field, instead of the incomprehensible abstract coloured gluons).

0.5.4 Hairy spaces, beaver spaces and fuzzy spaces

Nature raises the existence of objects, such as living organisms, whose anatomy suggests that mathematical objects having adjoined parts with each having different dimensions could exist. This can reflect the fact that the dimensions of some objects may even not be completely established. The existence of such strange objects implies that appropriate tools should be prepared for their eventual study [1].

A "hairy space" is a $(n > 3)$-ball having $1 - \mathrm{D}$ lines planted on it and for such spaces the volume and the area do not change with the insertion of 1-spaces on them.

Let we have a simplex S^n: the last segment allowing set E_{n+1} to be completed from E_n of S^{n-1} is (x_n, x_{n+1}). The simplex F^{n-1} such that one of its facets A^{n-2} can have its last segment $(y_{n-1}, y_{n-2}) \equiv (x_n, x_{n+1})$. Repeating this operation for descending values we get a space having n- and $(n-1)$-adjoined parts with their intersections having lower dimension: $\mathrm{Dim}\{S^n \cap S^{n-1}\} \leq (n-2)$. With respect to a beaver, having a spherical body, with a flat tail surrounded with hair, these spaces are denoted as "beaver spaces".

Beaver spaces implies some specific adjustment of the methods used for their scanning. They allow one to introduce a new mode of assessment of coordinates, consisting in studying the intersection of the unknown space with a probe composed of an ordered sequence of topological balls of decreasing dimensions, down to a point $(\mathrm{D} = 0)$. This process has been shown to have the advantage of being able to define coordinates even in a fractal space. This may be of particular interest with components provided by and embedded in the lattice $S(\emptyset)$.

Let simplex $S_n^n = \{(E_{n+}), \perp(n)\}$ have two characteristic structures $\mathbf{L}_n^1 = \sum_{j=1\to n}(\mathrm{dist}(x_{j-1}, x_j))$ and $\mathbf{Y}_{n+1}^1 = \sum_{j=1\to n}(\mathrm{dist}(x_{j+1}, x_j))$. Then, to estimate the simplicial dimension, a structure arises that defines the simplicial space in agreement with the (1 − D)-probe for the (D > 1)-space:

$$(\perp n) = \mathbf{Y}_{n+1}^1 > k(n,d)\cdot \mathbf{L}_n^1 \Leftrightarrow d > n. \quad (46)$$

If the last segment of the simplex (x_n, x_{n+1}) be such that $\mathrm{dist}(x_n, x_{n+1}) \in [0,\ 1]$, then the expression for $\mathbf{Y}_{n+1}^1$ reaches a value fuzzily situated between the assessment of $\mathrm{D} = n-1$ and $\mathrm{D} = n$ at least for $n < 4$. Thus, the simplicial set E_{n+1} is of the fuzzy type and the simplex is a fuzzy simplex.

This is the space with a "fuzzy dimension", and it provides a certain extension of the concept of a fuzzy set to the concept of a "fuzzy space of magma", since the set does not change, while this rule leads to a magma with a fuzzy structure. These problems could be the subject of further development, as similar things can be encountered when exploring the large scales of the Universe.

0.6 Concluding remarks

As we can see, today there is no holistic vision of the physical picture of the world.

Fractional charges on quarks are still considered a cornerstone of the Standard Model, and of course this ad-hoc hypothesis has not received honest and reliable experimental confirmation. Such excessive fantasy only spread the trend toward ever more abstract descriptions of real things and introduced deep confusion and misunderstanding into the physics community.

Alternative models proposing integer charged quarks do not fit within the framework of impressionistic physical mathematics and therefore do not find support among abstractly thinking particle theorists. However, only the integer value of the quark charge allows us to understand the process of neutrino production during hadron decay [10].

Basically, the Standard Model demonstrates a kind of refined science in which only speculative flows generated by physical mathematics are present. For example, the unification of incompatible disciplines such as symmetry and quantum mechanics – the former requires the object of study to remain rigid and unchanging during motion, while the latter, on the contrary, works not with the object, but with its operators and even more abstract ψ-function with an indefinite blurred position of the microscopic object. Such combinations lead to the theoretical generation of more and more new particles, new interactions and a new mind-blowing fantastic perception of the real world.

However, all these strange theoretical sets of fields and particles are actually just products of non-realistic mathematical combinations, which can be compared to the reflection of bizarre images in a kaleidoscope, behind which there are only a few real particles. The few particles mentioned are indeed the true primary objects that give rise to physical fields and composite particles. But how can such primary particles be found?

These and other heavy conceptual problems of particle physics noted above require solutions, and the solution lies in understanding the structure of real space from which all physics emerges.

The proposed mathematical approach is based on the idea that there must be a manifold that forms both objects and distances. And then the mathematical points turn into topological balls and the entire space becomes covered by these balls and they form the corresponding mathematical lattice. If such a lattice is a fractal lattice, then it is capable of generating physical space, and this fact has been demonstrated in the proposed approach, which originates from set theory, topology, and fractal geometry.

So, the concept of the tessellattice describes the universe at a submicroscopic level, far deeper than quantum mechanics, and derives from its principles such fundamental physical notions as mass and charge. Physical space presented as the tessellattice, which is a substrate, actually allows the propagation of real waves, so in this case the description of microphysical phenomena using the usual classical wave equation seems quite natural, and in this equation, the wave function ψ is nothing more than the mass density of the system $\{$particle + its inerton cloud$\}$ that moves in the tessellattice.

The tessellattice plays the role of neutrality, in which no physical parameter dominates over the others. However, neutrality can also be represented by symmetry and homogeneity of the distribution of convex and concave components simultaneously present on the same edges of cells, while the volume reduction associated to mass would remain fulfilled. The latter case would stand for a pseudo-neutrality worth to be taken in consideration.

Most examples of topological defects come from condensed matter physics, which is understandable, since the structure and geometry of condensed matter systems can be most carefully calculated. Cosmic scales can contain a huge number of various defects, but their study remain purely theoretical because researchers believe that in a cosmological context notably different physics must be involved, and it may be very distinct from terrestrial physics.

However, the discovery of a spatial structure in the form of the tessellattice allows us to take a different look at the mathematical basis that researchers lay when studying certain geometric, topological, or other mathematical features of various microscopic, terrestrial, or cosmic systems. The point is that the tessellattice itself is a type of condensed matter, and therefore its mathematical properties can be applied to physical systems of any scale. Physics of the tessellattice is the physics of massotension field and electromagnetic field (i.e., the inerton field and the photon field, respectively), given existing distributions of mass and charge. Thus, the study of any composite system, if accompanied by the tessellattice, will always have a solution or provide predictions that are as close as possible to reality.

Outside the class of leptons and the class of quarks, there are no more elementary particles; new particles have no way of appearing in principle.

The proposed approach to the development of physical and mathematical sciences should be successful, since it is confirmed both by clear mathematical constructions with direct access to mathematical physics, and by numerous experimental confirmations of the existence of inertons that are carriers of one of the two main fields of the Universe.

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