A DETAILED CONFUTATION OF LIGO'S STATEMENTS ON THE 150914-SIGNAL

Article · May 2016

READS
46

2 authors:

Angelo Loinger
University of Milan
133 PUBLICATIONS **909** CITATIONS
SEE PROFILE

Tiziana Marsico
LC "G. Berchet" Milan
38 PUBLICATIONS **65** CITATIONS
SEE PROFILE

April 20, 2016

A DETAILED CONFUTATION OF LIGO'S STATEMENTS ON THE 150914-SIGNAL

ANGELO LOINGER AND TIZIANA MARSICO

ABSTRACT. – "Much ado about nothing": as we demonstrate in the present paper, this is an adequate title of LIGO's article on the 150914-signal.

Consider an ensemble of gravitationally interacting particles. We have proved that their motions are *geodesic*; therefore, no gravitational wave (GW) is emitted. *A posteriori*, this result is intuitively evident: indeed, it is the analogue in a curved spacetime (the manifold created by the gravitating particles) of the obvious fact that in a flat Minkowskian spoacetime the trajectories of the free particles of an ensemble are rectilinear.

The undulatory solutions of the Einstein homogeneous field equations do not possess a true, generally covariant, energy-tensor, which is different from zero, *i.e.* they do not posses a *physical* reality. In general relativity (GR) the speeds of the reference frames are arbitrary, from zero to infinite; consequently, the same thing happens for the speeds of the undulatory metric tensors. The value c is not privileged, as the astrophysical community believes.

A historical remark. At the ends of the Thirties of past century, Einstein, Rosen, Infeld and other relativists had lost the belief in the real existence of the GWs. The Einstein-Infeld-Hoffmann method gave the key to find sceptical results. . –

A precise analysis of the metric tensor of Schwarzschild's manifold created by a gravitating mass-point allows to exclude the fictive properties which are ascribed to the locution "Black Hole" (BH); in particular, by virtue of Hilbert's gravitational repulsion, the BH cannot "swallow" anything.

A historical remark: all the Founding Fathers of general relativity rejected the fictive notion of black hole.

A distinguished physicist has pointed out that the stellar-mass BH candidates are in reality neutron stars inside massive accretion disks.

There exists no solution of the Einstein field equations concerning *two* interacting mass-points. Therefore, the higher-order post-Newtonian computations which pretend to describe the properties of a binary black hole (BBH), and the similar "approximate" numerical computations, are quite destitute of a mathematical and physical meaning. And we can affirm that no physicist has "seen" a BBH.–

The numerical data which "explain" the structure of the 150914-signal have been deduced by LIGO's collaborators from the mentioned "approximate" computations of invented BBHs and the GWs emitted by them. A very bold procedure:.

The reader of this long *Abstract* can judge the reliability of LIGO's work.

Date: April 20, 2016.

PACS 04.20 – General relativity.

Introduction. – In a recent concise Note [1], we have proved with essential arguments that LIGO's considerations on the 150914-signal [2] are meaningless, mathematically and physically. In order to reach a greater group of physicists, we give now a detailed confutation of LIGO's explanations of their results. Our paper is composed of a First Part (Non-existence of the GW's), a Second Part (Non-existence of a binary Schwarzschildian manifold of two mass-points), and an Appendix (Contra some considerations of the *Introduction* of [2]). It will be evident that a thing with the numerical properties of TABLE 1 in [2] does not belong to physical reality, but is a creation of senseless computations.

FIRST PART

1. – If we write, with Infeld [3], the Einstein field equations using tensor *densities*, and if

$$\delta(\mathbf{x}-\xi) =: \delta(x^1-\xi^1)\delta(x^2-\xi^2)\delta(x^3-\xi^3) \tag{1}$$

is the three-dimensional Dirac's generalized function, (ξ^1,ξ^2,ξ^3) being the space coordinates of a particle, we have for the mass-tensor density of an ensemble of s particles:

$$\mathcal{T}^{\mu\nu} = \sum_{p=1}^{s} \overset{p}{\mathcal{T}}{}^{\mu\nu} \quad , \tag{2}$$

$$\overset{p}{\mathcal{T}}{}^{\mu\nu} = \overset{p}{m}(t)\,\frac{\mathrm{d}\overset{p}{\xi}{}^{\mu}}{\mathrm{d}\tau}\,\frac{\mathrm{d}\overset{p}{\xi}{}^{\nu}}{\mathrm{d}\tau}\,\overset{p}{\delta}(\mathbf{x}-\xi) \quad . \tag{3}$$

Now, if the world lines of the particles never intersect, it is not difficult to verify that the differential equations of motion of the particles:

$$\sum_{p=1}^{s} \overset{p}{\mathcal{T}}{}^{\mu\nu}{}_{;\nu} = 0 \quad , \quad (\mu = 0,1,2,3) \quad , \tag{4}$$

where the semicolon denotes a covariant derivative, give the equations of *geodesic* lines [4]. B ut, as it is well known, a geodesic motion does not generate any wave.

The transition from a discrete to a continuous ensemble implies simply the substitution:

$$\mathcal{T}^{\mu\nu} \to \varrho(t,\mathbf{x})\,\frac{\mathrm{d}\xi^{\mu}(t,\mathbf{x})}{\mathrm{d}\tau}\,\frac{\mathrm{d}\xi^{\nu}(t,\mathbf{x})}{\mathrm{d}\tau}\,\sqrt{-g} \quad , \tag{5}$$

where ϱ is the mass density.

We see that the gravitational self-force theory, which is based on a conceptually wrong analogy with the electromagnetic self-force of a charge, does not make any sense. (This analogy is at the origin of the widespread *idée fixe* of the existence of GW's).

As we shall see in the sequel, in the *linearized* version of GR the particles of the ensemble describe *straight* lines in a Minkowskian spacetime.

2. – Another proof of the results of sect.**1** can be obtained in the following way. Let us consider a system of non-interacting particles which move in a Minkowskian spacetime. If $q^\mu(\tau), (\mu = 0, 1, 2, 3)$ are the spacetime coordinates of one of them as functions of the proper time τ, we have that

$$\mathcal{L}_{(0)} := \eta_{\mu\nu} \frac{\mathrm{d}q^\mu}{\mathrm{d}\tau} \frac{\mathrm{d}q^\nu}{\mathrm{d}\tau} = c^2 \tag{6}$$

is a first integral of Lagrange equations

$$\frac{\partial \mathcal{L}_{(0)}}{\partial q^\mu} - \frac{\mathrm{d}}{\mathrm{d}\tau} \frac{\partial L_{(0)}}{\partial \left(\mathrm{d}q^\mu/\mathrm{d}\tau\right)} = 0 \quad , \tag{7}$$

from which:

$$\frac{\mathrm{d}^2 q^\mu}{\mathrm{d}\tau^2} = 0 \quad , \tag{8}$$

i.e. a rectilinear and uniform motion.

Quite analogously, if we consider a system of particles interacting *only* gravitationally and moving in the Riemann-Einstein manifold created by them, we have that

$$\mathcal{L} := g_{\mu\nu}\left[q(\tau)\right] \frac{\mathrm{d}q^\mu}{\mathrm{d}\tau} \frac{\mathrm{d}q^\nu}{\mathrm{d}\tau} = c^2 \tag{9}$$

is a first integral of Lagrange equations

$$\frac{\partial \mathcal{L}}{\partial q^\mu} - \frac{\mathrm{d}}{\mathrm{d}\tau} \frac{\partial L}{\partial \left(\mathrm{d}q^\mu/\mathrm{d}\tau\right)} = 0 \quad , \tag{10}$$

which coincides with the *geodesic* equations

$$\frac{\mathrm{d}^2 q^\mu}{\mathrm{d}\tau^2} + \Gamma^\mu_{\varrho\sigma} \frac{\mathrm{d}q^\varrho}{\mathrm{d}\tau} \frac{\mathrm{d}q^\sigma}{\mathrm{d}\tau} = 0 \quad . \tag{11}$$

We see the revolutionary nature of GR, its conceptual base is radically different from that of Newton's gravitation. The particles of an ensemble move *freely* in a curved spacetime. Newton's force has been substituted by a geometrical property. This fact makes evident that no GW's can be generated.

3. – Consider a spherical continuous ensemble of particles, a spherical "cloud of dust". it is clear that the *rotation* of this body does not generate any GW.

4. – If a "cloud of dust" is composed of electrically charged particles – or of particles subjected to any field of force – the motions are not geodesic. However, no GW is created. This can be demonstrated and can be intuitively understood. as follows. The kinematical elements (velocity, acceleration, time derivative of the acceleration, *etc.*) of the motion of a particle in a given trajectory segment are identical to the analogous elements of the motion of a test-particle in a given gravitational field.

5. – In GR the speeds of the reference systems and consequently the speeds of the undulatory metrical tensors (mathematical solutions of Einstein equations $R_{\mu\nu} = 0$ – can have *any* value, from zero to infinite. The diffuse belief that any undulatory $g_{\mu\nu}$ is propagated with a velocity always equal to c is only an *idée fixe*.

Further, the wave nature of an undulatory $g_{\mu\nu}$ depends on the reference frame and can be destroyed by a suitable choice of general coordinates.

We see that the *general covariance* of GR-formalism implies not only the absence of body motions which create GWs, but also the arbitrariness of the wave nature and of the speed of any solution of the homogeneous Einstein's equations.

6. – A significant thesis by lorentz and Levi-Civita (which has been formally proved by one of us [5]) affirms that in Einstein field equations the matter tensor $T_{\mu\nu}$ is balanced exactly by $[R_{\mu\nu} - (1/2)g_{\mu\nu}\, R]\,/\kappa$, *which is the true gravitational energy-tensor*.

As Levi-Civita [6] emphasized, these facts have a momentous consequence: free waves and other purely gravitational phenomena are excluded. When $T_{\mu\nu}$ vanishes, the same must happen to the gravitational energy-tensor $[R_{\mu\nu} - (1/2)g_{\mu\nu}\, R]\,/\kappa$. "This fact entails a total absence of stresses, energy flow, and also of a simple localization of energy." [6].

7. – In 1930 Levi-Civita [7] demonstrated that the functions $z(x), [x \equiv (x^0, x^1, x^2, x^3)]$, of the characteristic hypersurfaces $z(x) = 0$ of Einstein field equations are solutions of the Hamilton-Jacobi equation

$$H := \frac{1}{2}\, g^{\lambda\mu}(x)\, \frac{\partial z(x)}{\partial x^\lambda}\, \frac{\partial z(x)}{\partial x^\mu} = 0 \quad . \tag{12}$$

According to Levi-Civita, the equation $z(x) = 0$ gives the law of motion of an *electromagnetic* wave-front: a natural extension of that valid in *special*

relativity. We see that also general relativity contains the basic law of the *geometrical optics*, and quite *independently* of Maxwell equations.

The characteristics of eq.(12) are electromagnetic world lines. Remark that both in GR and in SR – and *independently* of the theory of the characteristics – these null lines satisfy the condition $\mathrm{d}s = 0$, and are the solutions of the Hamilton equations:

$$(12bis) \qquad \frac{\mathrm{d}p_\lambda}{\mathrm{d}\sigma} = -\frac{\partial H}{\partial x^\lambda} \quad ; \quad \frac{\mathrm{d}x^\lambda}{\mathrm{d}\sigma} = \frac{\partial H}{\partial p_\lambda} \quad ; \quad \left(p_\lambda = \frac{\partial z}{\partial x^\lambda}\right) \quad ,$$

where σ is a convenient parameter.

8. – Any electromagnetic ray is a null geodesic in any spacetime; in particular, in the spacetime created by itself. Consequently, *no* undulatory, purely gravitational, and *autonomous* field is generated by the propagation of any electromagnetic wave in any spacetime manifold.

If the matter tensor $T_{\mu\nu}$ coincides with the electromagnetic energy-tensor $E_{\mu\nu}$, the Einstein equations have as a necessary consequence that the equation (12) is the equation of the characteristics of both Einstein *and Maxwell equations*.

9. – *Pseudo* (*i.e.* false) gravitational stress-momentum-energy tensor: a spurious notion, which has been formulated in various ways, and which has been unobjectionably rejected by Levi-Civita [6] with a stringent mathematical consideration. We have an object which is covariant only under *linear* coordinate-transformations, and that is *exactly* reducible to zero at any point of the spacetime manifold. Further, it can be created on a *flat* spacetime by using suitable curvilinear coordinates. Many authors have utilized it by assuming restrictions of its application domain, giving origin to hybrid, meaningless formulae, containing generally-covariant terms and terms which are covariant only under linear coordinate-transformations.

10. – *Linearized version – or linear approximation* – of GR. There are two methods of deduction, the usual one [8] and that of Weyl [9]. in the first, one starts from the approximate equality $g_{\mu\nu} \approx \eta_{\mu\nu} + \epsilon h_{\mu\nu}$, where ϵ is a small parameter, and one remarks that the symmetric field $h_{\mu\nu}$ is covariant only under the Lorentz transformations of the coordinates; $\eta_{\mu\nu}$ is the usual Minkowski tensor. Weyl [9] deduces the linearized version of GR quite *independently* of the Einstein field equations. He proves that there exists a *linear* field theory of gravitation in a Minkowskian spacetime, which has this gauge-invariance property:

$$(13) \qquad h^*_{\mu\nu} = h_{\mu\nu} + \frac{\partial \lambda_\mu}{\partial x^\nu} + \frac{\partial \lambda_\nu}{\partial x^\mu} \quad ,$$

where the four functions $\lambda_\mu(x)$ are arbitrary, and which gives the equations of the conventional procedure [8]. In this paper [9] Weyl makes a conceptually fundamental remark: the motions of the particles of a "cloud of dust" are Minkowskian geodetics, *i.e.* the four-velocities u^μ satisfy the equations $\mathrm{d}u^\mu/\mathrm{d}\tau = 0$: rectilinear and uniform motions. "From the standpoint of Einstein's theory this is as it should be, because *the gravitational force arises only when one continues the approximation beyond the linear stage*" [9]. (It is clear that this conclusion does not concern the geodesics of test-particles, or of light-rays, in a *given* approximate field $g_{\mu\nu} \approx \eta_{\mu\nu} + \epsilon h_{\mu\nu}$). Of course, the *approximate* field $h_{\mu\nu}$ can be quantized, but by virtue of the classical equations $\mathrm{d}u^\mu/\mathrm{d}\tau = 0$ no GW is emitted – the *graviton* is a science-fiction object. We remark finally that any reasonable program of an *exact* quantum gravity is doomed to failure [10].

11. – At the ends of the Thirties of past century, Einstein, Rosen, Infeld and other relativists had lost the belief in the real existence of the GWs. This fact is well known to the historians of physics. Thus, in the following years various sceptical considerations on the GWs were developed, in particular by Infeld and his pupil Scheidegger [3]. The Einstein-Infeld-Hoffmann method played a fundamental role. One develops all the functions that appear in the Einsteinian field equations into a power series of a small parameter λ; in particular, one puts:

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + \sum_{n=1}^{\infty} \lambda^n \underset{n}{h}_{\mu\nu}(x) \quad , \tag{14}$$

To find the motions of the point-masses of a discretized "cloud of dust" we have two perturbative approaches at our disposal: *i*) in the original EIH-approach one searches the solutions of $R_{\mu\nu} = 0$, in perfect analogy with the mass-point solutions of Laplace equation $\triangle U = 0$; *ii*) in Infeld's approach one follows the procedure of sect.**1**. (Remark that the employment of Dirac's delta-functions is analogous to their employment in Newton theory).

In the approximations higher than the second there are terms describing a gravitational-radiation damping. However, one can perform *at any stage* a suitable coordinate transformation which reduces them to zero. The equations of motion acquire a "Newton-like" form. This result is conceptually fundamental: it gives a significant corroboration of our *exact* result of sect.**1**. The objection that an "Ausstrahlungsbedingung" is not postulated is meaningless: in GR one can perform any continuous coordinate transformation.

12. – All the considerations of this First Part converge in the conclusion that the motions of the celestial bodies cannot give origin to GWs, because they are all *geodesic*. The diffuse belief that the accelerations – in particular, in collisions, in supernova explosions, *etc.* – can generate GWs is devoid of

any rational base: in GR the accelerations are destitute of any intrinsic value.

SECOND PART

13. – In 1926 Levi-Civita [11] gave a *geometrically explicit* explanation of the general form of solution (de Sitter, Eddington) to the Schwarzschild problem to find the Einsteinian field created by a gravitating point-mass M at rest. He adopted a Palatini's method [12], which yields the appropriate *geometrical definition of spherical symmetry in a curved spatial manifold*, and the justification of the employment in it of the polar coordinates $r(\geq 0)$, $\vartheta(0 \leq \vartheta \leq \pi)$, $\varphi(0 \leq \varphi < 2\pi)$. He found, with de Sitter and Eddington:

(15)
$$\mathrm{d}s^2 = \left[1 - \frac{2m}{\mathcal{R}(r)}\right] c^2\mathrm{d}t^2 - \left[1 - \frac{2m}{\mathcal{R}(r)}\right]^{-1} [\mathrm{d}\mathcal{R}(r)]^2 - [\mathcal{R}(r)]^2 \left(\mathrm{d}\vartheta^2 + \sin^2\vartheta\,\mathrm{d}\varphi^2\right) \quad ,$$

where: $m \equiv GM/c^2$, and $\mathcal{R}(r)$ is any regular function of r, which gives a Minkowskian $\mathrm{d}s^2$ at $r = \infty$. For $\mathcal{R}(r) = r$ we have the standard (Hilbert, Droste, Weyl) form of solution; for $\mathcal{R}(r) = [r^3 + (2m)^3]^{1/3}$ and $\mathcal{R}(r) = r + 2m$ the original Schwarzschild's [13] and Brillouin's [14] forms of solution, respectively. (Remark that the forms [13] and [14] are *maximally extended* – and thus the baroque known form of solution by Kruskal and Szekeres is quite superfluous.)

It is evident from Levi-Civita's treatment that eq. (15) has a mathematical and physical meaning *only for* $\mathcal{R}(r) > 2m$, and that no role inversion between $\mathcal{R}(r)$ and t for $\mathcal{R}(r) \leq 2m$ is allowed.

Temporarily forgetting that when $\mathcal{R}(r) \leq 2m$, eq. (15) loses any meaning, we could claim that the surface area $A = 4\pi(2m)^2$ represents an invariant and significant notion – and the so-called "Schwarzschild radius" $2m$ is physically meaningful. But this forgetting is not permitted, and we understand why the Founding Fathers of General Relativity (GR) rejected the idea to give a physical meaning to the "globe" $\mathcal{R}(r) \leq 2m$. As a matter of fact, the astrophysical phenomena that have been interpreted by a "globe" of this kind can be plainly interpreted as due to a great, or enormous, mass concentrated in a relatively small space region. In particular, no "swallowing" property of the *event horizon* $\mathcal{R}(r) = 2m$ has ever been observed. (Remark that the radially moving test-particles and light-rays arrive at $\mathcal{R}(r) = 2m$ with *zero* velocity and *zero* acceleration.)

Kundt [15] thinks that the stellar-mass "globe" candidates are in reality neutron stars inside massive accretion disks, and that the central engine of an AGN (active galactic nucleus) is a nuclear-burning disk.

A last remark. It was observed by von Laue [16] that the coordinate t of Schwarzschild manifold of a gravitating mass-point has a character of a *physical* "Systemzeit", as it is proved by its role in the explanations of the redshifts of the spectral lines.

14. – For further details about Schwarzschild manifold and its role in the gravitational collapses, see our paper "On the relativistic gravitational collapse *et cetera*" [17].

15. – There exists no solution of Einstein field equations concerning two interacting mass-points; in other terms, Schwarzschild's solution for a mass-point is not generalizable to a solution of two mass-points with their singularities.

Neglecting this fact, many authors have developed higher-order post-Newtonian "approximations" [18], or numerical "approximations" (see our paper [3]), that they think apt to describe the behaviour of two objects of the above kind, their orbit and their merger (as a consequence of the emission of GWs). One speaks currently of binary black holes (BBHs). We have avoided the term "black hole", because it is pregnant with fictive properties of the event horizon $\mathcal{R}(r) = 2m$, both theoretically and observationally [17].

16. – For the "explanation" of the data concerning the 150914-signal [Time: $(0 \div 45)$ s; Frequency: $(35 \div 250)$ Hz; "Peak strain": 1.0×10^{-21}] LIGO collaborators have extracted from the pseudo-demonstrations of GWs generated by invented BHHs, the numerical properties of TABLE 1: masses of the two BHs; mass of the final BH; spin of the final BH; luminosity distance (410 Mpc); source redshift $z = 0.09$. – The "explanation" of the final black hole spin is the following: "... the end product of a black hole binary coalescence is a Kerr black hole, which is fully described by its mass and spin." A totally false affirmation from the standpoint of the rigorous general relativity.

Conclusion. – We have given incontestable proofs of the non-existence of physical GWs and of binary Schwarzschildian manifolds created by two interacting mass-points. If, however, LIGO-collaborators persevere in giving credit to the existence of fictive celestial objects created by senseless "approximate" computations, they will have the possibility to "explain" the causes of various received signals. –

APPENDIX

The "Introduction" of LIGO's paper [2] contains some statements on past things that deserve a criticism. First of all, the current evaluation of the consequences of the discovery of the binary PSR1913+16.

As it was remarked in the detailed analysis of a paper of 2005 ([19]), the exact formulation of GR excludes the generation of physical GWs. Accordingly, the measured decrease of the orbital period of PSR1913+16 must

have other causes, different from the emission of GWs; maybe the viscous losses of the unseen pulsar companion, it it were, *e.g.* a helium star. We recall further that the pulsar is a "recycled" star, *i.e.* an object that was spun up by material accretion.

The computations that would explain this decrease have been developed using the *linear* approximation of GR. However, it is easy to prove the inadequacy of this approximation. Remember, *in primis*, the following Weyl's result (cf. sect. **10** of First Part): the particles of a "cloud of dust" must perform rectilinear and uniform motions; therefore, no GW is emitted.

In the well-known treatise by Landau and Lifshitz [20] we find a standard treatment of the linear approximation of GR, with a particular reference to the emission "mechanism" of GWs. The authors choose the matter tensor of a "cloud of dust", and with many clever simplifications and with the employment of the gravitational *pseudo* energy-tensor they arrive at the formula that gives the energy lost by the system in a time unit as a consequence of the emission of GWs. This result is meaningless; indeed: *i*) as we have seen, the elements of a "dust" follow rectilinear and uniform motions: *ii*) the gravitational *pseudo* energy-tensor is a spurious notion, extraneous to GR (cf. sect.**9** of First Part).

Conclusion: the measured rate of change of the orbital period of PSR1913+16 does not demonstrate the emission of GWs. –

Einstein and the GWs. The notion of GW has its origin in the linear approximation of GR, which was deduced by Einstein in 1916. As a consequence of the gravitational *pseudo* energy-tensor, the GW give rise to a dispersion of energy through irradiation. This result seemed unacceptable to Einstein, and it "should not have happened in nature." In the years of the American exile, Einstein and his co-workers arrive dat a negative conclusion about the real existence of the GWs. In *The Meaning of Relativity* (1955) no mention is made of Gws and BHs. –

A final remark. After the Chapel Hill Conference in 1957 the great majority of the theoretical astrophysicists became believers in the physical existence of the GWs. We think that scientific truth is not established by a majority, but only by Time. –

References

[1] A. Loinger and T. Marsico, "On LIGO's detection of a gravitational wave", *Academia.edu* and *Researchgate*, 13 February 2016.

[2] B. P. Abbott *et al.* (other 1003 co-authors), *Phys. Rev. Letters*, **116**, 061102 (2016).

[3] L. Infeld, *Acta Phys. Polonica*, **13** (1954) 187 – and the references therein. See also A. Scheidegger, *Revs. Mod. Phys.*, **25** (1953) 451; L. Infeld and J. Plebanski, *Motion and relativity* (Pergamon Press, Oxford, *etc.*) 1960.

[4] A. Loinger and T. Marsico, *arXiv:1211.6152* [physics.gen-ph] 18 Nov 2012.

[5] A. Loinger, *arXiv:1109.5268* [physics.gen-ph], 24 Sep 2011.

[6] T. Levi-Civita, *Rend. Lincei*, **26** (1917) 381; an English version in *arXiv:physics/9906004*, June 2nd, 1999.

[7] T. Levi-Civita, *Rend. Acc. Lincei*, **11** (s.6a), (1930), 3 and 113.

[8] See, *e.g.*, W. Pauli, *Teoria della Relatività* (Boringhieri ed., Torino) 1958, sect. **60**.

[9] H. Weyl, *Amer. J. Math.*, **66** (1944) 591.
[10] A. Loinger, *arXiv:physics/0308042* – August 12th, 2003.
[11] T. Levi-Civita, *The Absolute Differential Calculus (Calculus of Tensors)* – (Dover Publications, Inc., Mineola, N.Y.) 2005, Chapt. XII, sects. **11**, **12**, **13**. (First published by Blackie and Son Lim. – London and Glasgow, 1926).
[12] A. Palatini, *Nuovo Cimento*, **14** (1917) 12.
[13] K. Schwarzschild, *Berl.Ber.*, (1916) 196; an English version in *arXiv:physics/9905030*, May 12th, 1999.
[14] M. Brillouin, *Journ. Phys. Rad.*, **23** (1923) 43; an English version in *arXiv:physics/0002009* February 3rd, 2000.
[15] W. Kundt, *Astrophysics* (Springer-Verlag, Berlin, *etc.*) 2001, pp.190 and 187; and *passim*.
[16] M. v. Laue, *Die allgemeine Relativitätstheorie*, 4. neubearbeitete Auflage (Friedr. Vieweg und Sohn, Braunschweig) 1956, p.112.
[17] A. Loinger and T. Marsico, http://www.ilasl.org/index.php/Scienze/article/view/202/205, Oct. 23th, 2014 – See also: A. Loinger and T. Marsico,*arXiv:1011.2600* [physics.gen-ph] 11 Nov 2010.
[18] A. Loinger and T. Marsico, *arXiv:1205.3158* [physics-gen-ph] 13 May 2012.
[19] A. Loinger, *arXiv:physics/0502089* – February 16th, 2005.
[20] L. Landau et E. Lifchitz, *Théorie du Champ* (Éditions MIR, Moscou) 1966, sect. **104**.

A.L. – Dipartimento di Fisica, Università di Milano, Via Celoria, 16 - 20133 Milano (Italy)

T.M. – Liceo Classico "G. Berchet", Via della Commenda, 26 - 20122 Milano (Italy)

E-mail address: angelo.loinger@mi.infn.it
E-mail address: martiz64@libero.it